Coherent states for anharmonic potentials

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Coherent states for the harmonic oscillator were introduced in 1963 by Glauber[1] and have been widely used in quantum optics. They are such that: (1) they minimize the uncertainty relation $(\Delta x)^2(\Delta p)^2 \geq \hbar^2/4$, (2) they are eigenstates of the annihilation operator of the harmonic oscillator $a|\alpha\rangle = \alpha|\alpha\rangle$, (3) they are created from the ground state by a unitary displacement operator $\exp(\alpha a^\dagger - \alpha^* a)|0\rangle = |\alpha\rangle$. These properties are equivalent for the coherent states of the harmonic oscillator, they are not so for the case of general potentials [2].

Let us introduce creation and annihilation operators $b^\dagger$, b as a renormalization of the usual SU(2) generators through [3]:

$$b^\dagger = \frac{J_-}{\sqrt{N}}, \quad b = \frac{J_+}{\sqrt{N}}, \tag{1}$$

and define the diagonal operator $\nu \equiv \frac{N}{2} - J_0$.

Using the commutation relations $[J_0, J_\pm] = \pm J_\pm, \quad [J_+, J_-] = 2J_0$, we obtain:

$$[b, b^\dagger] = 1 - \frac{2\nu}{N}, \quad [\nu, b] = -b, \quad [\nu, b^\dagger] = b^\dagger \tag{2}$$

which are similar to the commutation relations for the usual harmonic oscillator except for the ν dependent term. A Hamiltonian of the form

$$H = \frac{\hbar\omega_0}{2}(b^\dagger b + b b^\dagger)$$

is diagonal in the $|J, m\rangle = |N, \nu\rangle$ basis with eigenvalue

$$E = \hbar\omega(\nu + \frac{1}{2}) - \frac{\hbar\omega}{N+1}(\nu + \frac{1}{2})^2 - \frac{\hbar\omega}{4(N+1)}, \quad \nu = 0, 1, \ldots, [N/2].$$

where $\omega = \frac{N+1}{N}\omega_0$. Except for the constant term, this is the Morse spectrum [4]. Now let us construct the state

$$|\beta\rangle = \exp(\beta b^\dagger - \beta^* b)|N, 0\rangle$$

which corresponds to the usual coherent state in the limit of large N. Writting the operators b, $b^\dagger$ in terms of the J's we get:

$$|\beta\rangle = \left[\cos^2\left(\frac{|\beta|}{\sqrt{N}}\right)\right]^J \sum_{m=-J}^{J} \left(\begin{array}{c} 2J \\ J+m \end{array}\right)^{1/2} \left[\frac{|\beta|}{\beta^*}\tan\left(\frac{|\beta|}{\sqrt{N}}\right)\right]^{J+m} |J, -m\rangle \tag{3}$$

which has the same form as the atomic coherent state introduced by Arrechi et. al. [5].

Consider the f-oscillators introduced by Man'ko [6] as a generalization to the q-oscillators [7] which can be interpreted as nonlinear oscillators with a very specific type of nonlinearity. Let the operators A and $A^\dagger$ represent the dynamical variables to be associated with the quantum f oscillators [8].

$$A = af(\hat{n}) = f(\hat{n}+1)a, \quad \hat{n} = a^\dagger a \quad A^\dagger = f(\hat{n})a^\dagger = a^\dagger f(\hat{n}+1) \tag{4}$$

where $f(\hat{n})$ is an operator-valued function of the number operator. The commutation relations for the operators A, $A^\dagger$ are

$$[A, A^\dagger] = (\hat{n}+1)f^2(\hat{n}+1) - \hat{n}f^2(\hat{n}), \quad [\hat{n}, A] = -A, \quad [\hat{n}, A^\dagger] = A^\dagger \tag{5}$$

If we choose $f^2(\hat{n}) = 1 + \frac{1}{N}(1-\hat{n})$, then, $[A, A^\dagger] = [b, b^\dagger] = 1 - \frac{2\nu}{N}, \quad \nu \equiv \hat{n}$.

We now construct a coherent state of the above f-deformed algebra as an eigenstate of the annihilation operator A

$$A|\alpha, f\rangle = \alpha|\alpha, f\rangle. \tag{6}$$

The representation of this state in the number basis can seen to be [6]:

$$|\alpha, f\rangle = N_f \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}f(n)!}|n\rangle \quad \text{with} \quad N_f = \left(\sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n![f(n)!]^2}\right)^{-1/2}$$

and the convention $f(n)! = f(0)f(1)\ldots f(n)$. Substitution of $f^2(\hat{n})$ gives

$$|\alpha, f\rangle = N_f \sum_{n=0}^{N} \sqrt{\frac{n^{n+1}}{n!(n+1)!}}\alpha^n|n\rangle \tag{7}$$

with the normalization constant

$$N_f = \left[\sum_{n=0}^{N} \frac{|\alpha|^{2n}n^{n+1}}{n!(n+1)!}\right]^{-1/2}.$$

The above states have been constructed with the condition of being eigenstates of the annihilation operator, how do these states compare with those obtained with the displacement operator acting upon the extremal state?

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